

YITP-SB-00-66
BNL-HET-00/40

QCD THEORY AT HIGH ENERGY ¹

George Sterman

*C.N. Yang Institute for Theoretical Physics, State University of New York at Stony Brook
Stony Brook, NY 11794-3840 USA*

*Physics Department, Brookhaven National Laboratory
Upton, NY 11973, USA*

Abstract

The energy range and quality of strong-interaction data from recent years demand the study of higher orders in perturbative QCD, and of nonperturbative effects. I discuss a selection of recent progress in the theory of QCD at high energy, including examples from perturbative resummation, nonperturbative power corrections and the Regge limit. In each case, techniques of factorization play a central role.

1 Introduction

Current studies in QCD are motivated partly by its importance in the production and detection of new physics, but also, and in very large part, by the challenges of quantum chromodynamics itself. QCD may be thought of as an exemplary quantum field theory, exhibiting asymptotic freedom, confinement, chiral symmetry breaking and so on, the workings of which are all available, and unavoidable, within present energy ranges. It is a vast subject, encompassing perturbative, heavy-quark, nonrelativistic and lattice QCD, all the way to nuclear physics. In a very real sense, what are sometimes called “tests of QCD” are tests of quantum field theory itself.

¹Based on a talk presented at the *IVth Rencontres du Vietnam, International Conference on Physics at Extreme Energies*, Hanoi, Vietnam, July 19-25, 2000.

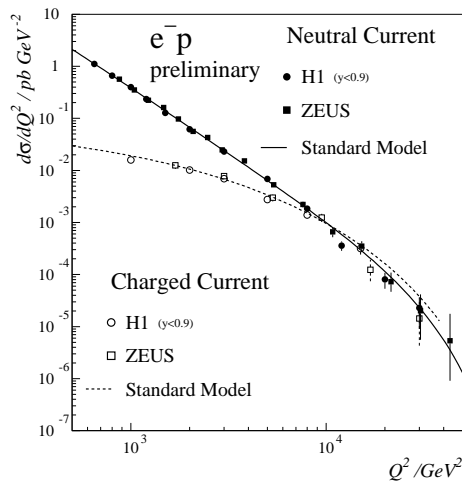


Figure 1: DIS cross section for neutral and charged currents at high momentum transfers. [1]

In this talk, I will discuss some recent efforts to narrow the gap between the high-energy, partonic and low-energy, hadronic descriptions of QCD, starting from the high-energy side, through the study of higher-order corrections in perturbation theory and of nonperturbative power corrections. I will highlight the use of QCD factorization as an organizing principle in these investigations.

The past decade has been a proverbial golden age of hadronic data at high energy, in terms both of coverage and of quality. There is no room here to do justice to the data revolution of the 1990's, the work of a generation of accelerators that has reached maturity: LEP, HERA and the Tevatron. Each has produced spectacular successes for our current picture of QCD, but each has provided its share of puzzles. Two examples must suffice. Our control of strong interaction corrections in inclusive cross sections such as deep-inelastic scattering (DIS) may be gauged from Fig. 1, showing data from HERA. [1] Here, the total ep cross sections at momentum transfers from tens into hundreds of GeV track the standard model predictions, which include extensive input from perturbative QCD evolution, described below.

On the other hand, Fig. 2 shows the transverse momentum distribution for b-quark production at the Tevatron. [2] This data is typical of cases where our present theory is partly adequate, partly not. Overall, the theoretical p_T spectrum has the correct shape, but the normalization of the theory is too low at low p_T , even as it gradually approaches the data at larger p_T . Much of the effort described below is aimed at using such apparent discrepancies as guides to the perturbative and nonperturbative structure of the theory. To see how, let us briefly review the elements of our present theoretical framework for high-energy QCD, based on factorization.

2 The Unity of QCD Factorizations

The application of perturbation theory to high energy QCD begins with asymptotic freedom and infrared safety, illustrated by the perturbative expansions of the total cross sections for e^+e^-

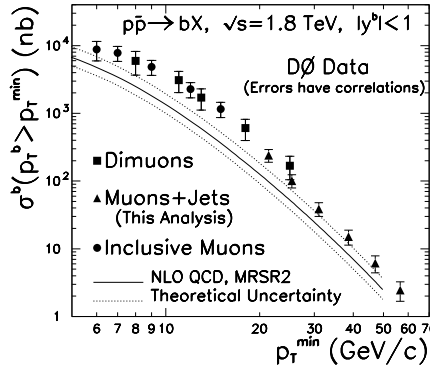


Figure 2: Transverse-momentum distribution for b-quark production at the Tevatron.[2]

annihilation to hadrons, and to final-state jets:

$$Q^2 \sigma(Q^2, \mu^2, \alpha_s(\mu)) = \sum_n c_n(Q^2/\mu^2) \alpha_s^n(\mu) + \mathcal{O}(1/Q^p) = \sum_n c_n(1) \alpha_s^n(\mu) + \mathcal{O}(1/Q^p), \quad (1)$$

where the c_n are dimensionless coefficients. For an infrared safe cross section, the c_n are free of dependence on fixed mass scales (such as light quark masses), which are absorbed into corrections that are suppressed by some power, p , of the c.m.s. energy, Q . In most cases, the c_n are fully known only for a few, low orders. Because the cross section is a physical observable, it is independent of the renormalization scale, μ , which can therefore be chosen to equal Q . For an asymptotically free theory, the larger is Q , the better any finite-order approximation becomes.

Relatively few cross sections are quite this simple, however, but whenever a reaction involves a scattering at large momentum transfer, or the creation or decay of a heavy state, we may isolate its short-distance components, which can be treated perturbatively, from its long-distance, nonperturbative components. This is a procedure known as *factorization*, which generalizes the operator product expansion.

For a factorized cross section, Eq. (1) is replaced by an expression of the general form,

$$Q^2 \sigma(Q, x) = \omega(Q/\mu, x/\xi, \alpha_s(\mu)) \otimes f(\xi, \mu) + \mathcal{O}(1/Q^p), \quad (2)$$

with a “hard-scattering”, or coefficient, function ω , which is short-distance and perturbative, in convolution with a “soft” function f , which is long-distance and nonperturbative. In DIS, with q the momentum transfer, $x = 2p \cdot q/Q^2$, but more generally it represents any dimensionless ratios of large momentum scales. The dimensional variable μ is the factorization scale, separating long and short distances. As in (1), the physical cross section is independent of μ . In DIS, the soft function f is a parton distribution function (PDF). For hadron-hadron scattering, we have two PDF’s in convolution form. In these cases, the convolution in (2) is in terms of fractional momenta, ξ , of the (one or more) partons that initiate the hard-scattering process. Factorization is more general than this, however, and we shall encounter other examples below. Nearly always, the soft function can be interpreted as the matrix element of some (usually nonlocal) operator in QCD.

The basis of factorization is always the quantum-mechanical incoherence of dynamics at very short distances from that at long distances, and, in Minkowski space, the mutual incoherence of the dynamics of particles whose relative velocity approaches the speed of light.

Whenever there is factorization, there is *evolution*, a consequence of the independence of the physical cross section from the factorization scale,

$$\mu \frac{d}{d\mu} \ln \sigma(Q, x, m) = \mu \frac{d}{d\mu} \ln \{ \omega(Q/\mu, x/\xi, \alpha_s(\mu)) \otimes f(\xi, \mu) \} = 0. \quad (3)$$

Because f and ω have in common only the parton momentum fraction and α_s , separation-of-variable arguments imply complementary equations for f and ω :

$$\mu \frac{df(\xi, \mu)}{d\mu} = P(z, \alpha_s) \otimes f(\xi/z, \mu) \quad \text{and} \quad \omega(Q/\mu, \eta z, \alpha_s) \otimes P(z, \alpha_s) = -\mu \frac{d\omega(Q/\mu, \eta, \alpha_s)}{d\mu}, \quad (4)$$

in terms of convolutions with splitting functions $P(z, \alpha_s)$. The first of these “DGLAP evolution” relations enables us to take PDFs determined at some reference scale, μ_o , and extrapolate to higher, or lower, scales, wherever the running coupling is not too large.

Measurements of the strong coupling based on these methods give $\alpha_s(M_Z) \sim 0.12$, which suggests that at around 100 GeV, $\mathcal{O}(\alpha_s^2)$ is about one percent. This is the nominal level of accuracy to which perturbative QCD may aspire at “next-to-next-to-leading” (NNLO) order. Referring to Eq. (1), the c_n ’s are known to NLO ($n = 1$) for cross sections with up to four jets in e^+e^- . [3] To date, complete NNLO calculations are available only for one-scale problems: the total e^+e^- cross section, DIS and Drell-Yan. Two loops are the current frontier for finite-order perturbative QCD, and the past year has seen significant progress toward the exact computation of two-loop scattering amplitudes and coefficient functions ω . [4] At the same time, to use two-loop coefficient functions, it will be necessary to have the splitting functions P at *three* loops; and here also important progress has been reported within just the past few months. [5, 6]

Beyond exact calculations, DGLAP evolution is the first among a set of methods that enable us to probe properties of QCD perturbation theory to all orders. Each of these methods is based upon the separation of dynamics at different length scales.

3 Resummation for Inclusive and Exclusive Cross Sections

Hard-scattering, such as jet, heavy quark and high- p_T photon production, depends on a complex combination of the long-distance dynamics of the external hadrons, the short-distance perturbative subprocess and the properties of final states included in the cross section. Using concepts of factorization, we are learning to treat a widening variety of such cross sections, and increasingly to compute classes of higher-order corrections. We describe below three applications of current interest.

3.1 Partonic Threshold

Our first example is so-called threshold resummation, which applies to inclusive hard-scattering hadronic cross sections $AB \rightarrow F + X$, with $F = \gamma^*, W, Z$, jets, heavy quark, etc. of invariant mass Q . We are interested in higher-order corrections to the perturbative hard-scattering function $\omega_{ab \rightarrow F}$ in factorized cross sections, Eq. (2), at *partonic threshold*, $z = Q^2/\hat{s} \rightarrow 1$, where the partons a and b have just enough c.m.s. energy $\sqrt{\hat{s}}$ to produce the observed final state. In the threshold region, there is an incomplete cancellation of emission and virtual radiative corrections, which leads to

singular corrections of the form

$$\omega_{ab \rightarrow F}^{(r)}(z) \sim \left(C_a \frac{\alpha_s}{\pi}\right)^r \frac{1}{r!} \left[\frac{\ln^{2r-1}(1-z)}{1-z} \right]_+ \quad (5)$$

at r th order, with $C_q = C_F$, $C_g = C_A$. In effect, for $z \rightarrow 1$, there are two hard scales, Q and $(1-z)Q$. Because these singular distributions arise from soft-gluon radiation, sensitive to the scale $(1-z)Q$, but not to the scale Q , they may be “refactorized” from the hard-scattering into functions that depend only on the flow of color at a truly short-distance hard-scattering, which is sensitive only to Q . As for Eq. (2), the new factorization implies a new evolution equation, from which we derive a resummation in moment space,

$$\begin{aligned} \tilde{\omega}_{ab \rightarrow F}(Q/\mu, N) &\equiv \int_0^1 dz z^{N-1} \omega_{ab \rightarrow F}(Q/\mu, z) \\ &= \exp \left[- \int_0^1 dz \frac{z^{N-1} - 1}{1-z} \int_{(1-z)^2 Q^2}^{\mu^2} \frac{dm^2}{m^2} A_{ab}(\alpha_s(m)) \right], \end{aligned} \quad (6)$$

where $A_{ab} = (C_a + C_b)(\alpha_s/\pi) + \dots$ is an expansion in α_s .

Threshold resummation is currently being explored for most of the basic inclusive cross sections. [7, 8] An important consequence is a reduction of factorization-scale dependence in resummed cross sections. In NLO calculations, dependence on μ begins at the next order, $\mathcal{O}(\alpha_s^2)$, but often this residual sensitivity is uncomfortably large. In the resummed function Eq. (6), however, the μ -dependence is determined by

$$\frac{d \ln \tilde{\omega}_{ab}(N, \mu)}{d \ln \mu} \sim A_{ab}(\alpha_s) \ln N \sim - \frac{d \ln [\tilde{f}_{a/A}(N, \mu) \tilde{f}_{b/B}(N, \mu)]}{d \ln \mu}, \quad (7)$$

where to the right the \tilde{f} ’s are moments of the parton distributions. Factorization requires that the function A_{ab} that appears in Eq. (6) is exactly the same as the sum of the $\ln N$ terms in the moments of the splitting functions P_{aa} and P_{bb} . This leads to a very significant decrease in sensitivity to the factorization scale compared to previous, fixed order calculations. [10, 11] This is only the beginning of applications of threshold resummation, however, and we anticipate important applications to the determination of parton distributions and to the improvement of predictions for new particle production.

3.2 Power Corrections: Universality and Beyond

Jet cross sections in e^+e^- annihilation are defined by adjustable parameters, whose variation mediates between fully inclusive and nearly exclusive cross sections. As such, they are ideal for testing and improving our understanding of QCD at intermediate distances.

The most-studied examples involve light-mass dijet pairs. Dijet events, which dominate the annihilation cross section at high energy, can be described in terms of event shapes. Perhaps the best known of these is the thrust, T . The thrust of an e^+e^- event is determined approximately by finding an axis that maximizes the quantity $T = 1 - (m_1^2 + m_2^2)/Q^2$, where m_1 and m_2 are the invariant masses of the sums of all particle momenta within the two hemispheres defined by this axis. As $T \rightarrow 1$, the final state is characterized by two well-collimated jets. A number of other familiar event shapes may be derived from jet masses in a similar way.

For light-mass dijets, the relative velocities of the two jets again insures that their dynamical developments into the final state are mutually independent, This independence results in factorization at the level of cross sections, and leads, in much the same manner as above, to evolution equations, and to T -moments that are quite similar to the z -moments of threshold resummation in Eq. (6),

$$\int dT T^{N-1} \frac{d\sigma_{\text{PT}}(T)}{dT} \sim \sigma_{\text{tot}} \exp \left[- \int_0^1 dy \frac{y^N - 1}{1 - y} \int_{(1-y)^2 Q^2}^{(1-y)Q^2} \frac{dk_T^2}{k_T^2} A_{q\bar{q}}(\alpha_s(k_T)) \right]. \quad (8)$$

Such resummed cross sections improve the perturbative description of differential cross sections, $d\sigma_{\text{PT}}/de$, for a class of event shapes e , including the thrust. [12] Nevertheless, for small $1 - T$, that is, close to the limit of massless jets, fits based on Eq. (8) fail to describe the data. This is not surprising, because the lighter the jets, the more dependent on long times are their cross sections, and correspondingly the more sensitive they are to nonperturbative effects. The mass of the jet is a “dial” for tuning the importance of nonperturbative dynamics.

Although perturbation theory cannot predict true long-time behavior, it can give hints as to its nature. In Eq. (8) these hints come from the running coupling. When the variable y gets close enough to one, the running coupling in the integrals in (8) diverges, signalling a breakdown of perturbation theory. This divergence is associated with a region of fixed size in k_T , independent of N . We should think of this as an ambiguity in perturbation theory, which is resolved in the full theory by nonperturbative information. [13, 14] Since the perturbative range of integration in Eq. (8) should remain meaningful, the minimal modification necessary to (8) is to replace the lower limit of the k_T -integral with a nonperturbative parameter: α_0 , with

$$\alpha_0 \equiv \frac{1}{\mu_0} \int_0^{\mu_0} dk \alpha_s(k), \quad (9)$$

where μ_0 is a conveniently chosen cutoff. The quantity α_0 has the interpretation of the integral of the running coupling over the nonperturbative region.

Letting $e \equiv 1 - T$, we can invert the transform (8). The substitution (9) then produces a simple shift in the perturbative spectrum, [15]

$$\frac{d\sigma(e)}{de} = \frac{d\sigma_{\text{PT}}(e - \lambda_e/Q)}{de} + \mathcal{O}\left(\frac{1}{e^2 Q^2}\right), \quad (10)$$

where we note that the relative size of the effect is λ_e/eQ , and that corrections begin at $1/(eQ)^2$. Similar considerations apply to any event shape e that vanishes in the limit of light-like dijets.

This approach has been formalized and applied in Refs. [16]. The integral of the running coupling, Eq. (9), is thought of as a fundamental, universal parameter. Applications, which include the partial incorporation of higher-order perturbative terms, provide an improved picture of differential event shapes, and a unified description of low moments of $e = 1 - T \dots$, such as $\int dT (1 - T) d\sigma/dT$. In addition, the approximation relates first to second moments:

$$\langle e^2 \rangle = \langle e^2 \rangle_{\text{PT}} + 2 \frac{\lambda_e}{Q} \langle e \rangle_{\text{PT}} + \frac{\lambda_e^2}{Q^2}. \quad (11)$$

This description has been reasonably successful in tying together the first moments of different event shapes. [17] Second moments, however, show large $1/Q^2$ corrections, compared to the predictions of strong-coupling universality, Eq. (9). [18]

A more general formalism involves the introduction of nonperturbative “shape” functions [19] that generalize Eq. (10) to a distribution of shifts due to soft radiation,

$$\frac{d\sigma}{de} = \int_0^{eQ} d\epsilon f_e(\epsilon) \frac{d\sigma_{\text{PT}}(e - \epsilon/Q)}{de} + \mathcal{O}\left(\frac{1}{eQ^2}\right). \quad (12)$$

The most important features of this expression are the level of the corrections, down by a full power of Q compared to Eq. (10), and the independence of the shape function f_e from Q . The latter implies that a fit to f_e at one value of Q , most conveniently $Q = m_Z$, is sufficient to predict the differential cross section for all Q . [19]

Equation (12) may be derived from the factorization properties of soft radiation in perturbation theory, and the shape function itself has the interpretation of a matrix element in QCD. To be specific, in the case of thrust the matrix element is

$$f_{1-T}(\varepsilon, \mu_{\text{IR}}) = \langle 0 | W^\dagger(0) \delta\left(\varepsilon - \int d\vec{n} (1 - |\cos\theta|) \mathcal{E}(\vec{n})\right) W(0) | 0 \rangle_{k_\perp < \mu_{\text{IR}}}, \quad (13)$$

with θ the angle between \vec{n} and the thrust axis. We define the operators W in terms of path-ordered nonabelian phase operators,

$$W(0) = P \, e^{ig \int_0^\infty d\lambda \beta \cdot A(\lambda\beta)} \left[P \, e^{ig \int_0^\infty d\lambda \beta' \cdot A(\lambda\beta')} \right]^\dagger, \quad (14)$$

and the operators \mathcal{E} are defined to measure energy flow, by [20]

$$\mathcal{E}(\vec{n}) | N \rangle \equiv \sum_{i=1}^N \delta(\cos\theta - \cos\theta_i) \delta(\varphi - \varphi_i) E_i | N \rangle, \quad (15)$$

for any final state with N particles. The matrix element in Eq. (13) is matched to perturbation theory by a cutoff in the transverse momentum of the soft radiation at μ_{IR} . In this formulation, true universality resides at the level of correlators of energy flow in the presence of the color sources,

$$\mathcal{G}(\vec{n}_1 \dots \vec{n}_L; \mu) = \langle 0 | W^\dagger(0) \mathcal{E}(\vec{n}_1) \dots \mathcal{E}(\vec{n}_L) W(0) | 0 \rangle. \quad (16)$$

A “mean field approximation”, which eliminates nontrivial correlations between measurements of energy flow in different directions:

$$\mathcal{G}(\vec{n}_1 \dots \vec{n}_L; \mu) \rightarrow \prod_{i=1}^L \mathcal{G}(\vec{n}_i, \mu) \quad (17)$$

reduces the shape function in (12) to a delta function, $f_e(\epsilon) \rightarrow \delta(\epsilon - \lambda_e)$. In this approximation, the shift (10) is recovered.

We may take another viewpoint of Eq. (13), and interpret it as a matrix element in an effective theory for soft radiation from light-like color sources. This is a natural language for the community with special interest in inclusive B decay near the edge of phase space, i.e., with jet-like final states. [21] In this case, the color source is the light quark that emerges from the decay $b \rightarrow s\gamma$, for example, whose bremsstrahlung factorizes from the remainder of the process.

3.3 Exclusive B Decay

Our third example is the recent application of factorization to the fully exclusive B decays into two mesons, M_1, M_2 , either heavy-light ($D\pi$), or light-light ($\pi\pi, K\pi$). Because the incoming quark is heavy, the appropriate factorization is somewhat different than in the previous examples, but the light-like relative velocity of the light meson(s) in the final state leads to a factorized form for the decay amplitude \mathcal{A} (here for the light-light case): [22, 23]

$$\begin{aligned} \mathcal{A}(B \rightarrow M_1 M_2) = & F_{B \rightarrow M_1} \hat{T}^I(m_b, \mu) \otimes \Phi_{M_2}(\mu) \\ & + \hat{T}^{II}(m_b, \mu) \otimes \Phi_B(\mu) \otimes \Phi_{M_2}(\mu) \otimes \Phi_{M_1}(y, \mu), \end{aligned} \quad (18)$$

where the short-distance functions \hat{T} are computable in perturbation theory, while the Φ 's are nonperturbative matrix elements that are wave functions for the hadrons. Further nonperturbative information is contained in $F_{B \rightarrow M_1}$, which is itself a matrix element. It may be possible to compute this matrix element if transverse momenta are included in the convolution. [23] Excitement has been generated by the possibility of using the formalism to isolate weak, CP-violating phases in these decays. This should be possible because all strong-interaction phases are contained in the functions \hat{T} , to leading power in m_b . We still have things to learn about the relationship between the different approaches to this factorization, and about the important role of power corrections. Nevertheless, the extension of factorization methods to this class of physical problems, whose interest transcends QCD, is an important step forward.

4 BFKL and High Parton Density

Everything we've discussed so far has involved hard scattering, and hence is restricted to rare processes. In recent years, however, considerable attention has returned to the bulk of the high energy cross sections, involving relatively low momentum transfers at high energy, including the Regge limit ($s \rightarrow \infty$ with t fixed), diffractive scattering and the total cross section. This classic constellation of topics is coming to the fore once again, in the light of the copious HERA data on small- x DIS, and renewed progress in the perturbative description of the total cross section, the so-called "perturbative pomeron" of QCD, as described by the celebrated BFKL equation.

4.1 BFKL 2000

The BFKL equation, with the LO kernel shown explicitly, can be written as

$$\xi \frac{d\psi(\xi, k_T)}{d\xi} = -\frac{\alpha_s N}{\pi^2} \int \frac{d^2 k_T}{(k_T - k'_T)^2} \left[\psi(\xi, k_T) - \frac{k'_T{}^2}{2k_T^2} \psi(\xi, k'_T) \right] + \text{NLO}. \quad (19)$$

The stimulus for much recent work is the newly-calculated explicit form of the NLO kernel, the fruit of a decade of effort. [24]

As shown in [25], Eq. (19) may be derived from a factorization characteristic of the Regge limit. [26] To be specific, we consider the forward scattering amplitude for virtual-photon-proton scattering, which is related by the optical theorem to the structure functions of DIS. At low x , we need only include (color singlet) gluon exchanges in the t -channel, or equivalently only the gluon distribution $G(x, Q)$. Introducing the "unintegrated" gluon distribution $\psi(\xi, k_T)$ through

$$G(\xi, Q) = \int^Q d^2 k_T \psi(\xi, k_T), \quad (20)$$

the relevant factorization for structure function F is

$$F(x, Q^2) = \int d^2 k_T c\left(\frac{x}{\xi}, Q, k_T\right) \psi(\xi, k_T) \left(1 + \mathcal{O}\left(\frac{1}{\ln^2 x}\right)\right), \quad (21)$$

in terms of a modified coefficient function, c . Relative to Eq. (2), the roles of k_T and ξ have been exchanged: ξ is now the factorization scale, separating, in the terminology of [25] “fast” from “slow” quanta, and k_T is now the convolution variable. On the one hand, the incoherence of the dynamics of fast and slow quanta make it possible to factorize the amplitude; on the other hand, Lorentz invariance leaves the division between the two arbitrary. This arbitrariness leads to an evolution equation, the BFKL equation. We note, however, that the factorization in Eq. (21) holds to next-to-leading logarithm in $x \sim s/Q^2$ only. Beyond this level, we must generalize the equation itself.

The ansatz $\psi \sim x^{-\omega} (k_T^2/\mu^2)^{\gamma-1}$, in Eq. (19), gives a consistency equation relating the exponents ω and γ . With $\bar{\alpha}_s \equiv N_c \alpha_s/\pi$, this is

$$\omega(\gamma) = \bar{\alpha}_s \chi_0(\gamma) \left[1 - \beta_0 \frac{\alpha_s}{4\pi} \ln \frac{k^2}{\mu^2} \right] + \bar{\alpha}_s^2 \chi_1(\gamma), \quad (22)$$

where the function $\chi_0(\gamma)$ has long been known, and where χ_1 is new, and the subject of much investigation.

The largest value of ω gives the dominant small- x , or equivalently large- s behavior. [24] Exhibiting only the LO result in analytic form, one finds

$$\omega_{\max} = 4N_c \ln 2 (\alpha_s/\pi) [1 - 6.5\bar{\alpha}_s] \Rightarrow \psi \sim s^{4N_c \ln 2 (\alpha_s/\pi) - \text{large}}. \quad (23)$$

From LO in the kernel we have QCD Regge behavior, but the innocuous-looking NLO result is, as it stands, not quite acceptable. It is simply too large and negative, and can eventually lead, not only to a decrease with s , but even to negative cross sections. This produced a bit of initial consternation on the part of some enthusiasts, but, ever-resourceful, investigators have developed very plausible proposals on how to proceed. In fact, the problem may be traced to “collinear divergences” in $\chi_1(\gamma)$, which, from the limits $k_T \rightarrow 0$ and $k_T \rightarrow k'_T$ in Eq. (19), receives poles up to γ^{-3} and $(1-\gamma)^{-3}$. [27]

Proposals on how to interpret the NLO kernel have included: (1) Adjust the scale of α_s , [28] (2) Impose kinematic constraints in \mathcal{K} , demanding strong ordering of particles in rapidity; [29] (3) Import information from DGLAP evolution, given the association of collinear logarithms in DIS to the singular behavior of χ . [30] Particularly for the latter proposals, the connection of BFKL to small- x DIS may suggest phenomenological tests of their efficacy. This story is probably just beginning.

4.2 Effective theories and high parton density

Sometimes it can be difficult for those not working on small- x and BFKL to appreciate fully their perennial fascination. One way of looking at what’s special about BFKL evolution is that, if the LO BFKL equation is not too misleading, then as we evolve to low x we are forced to a regime of high parton density even at “fixed” (actually diffusing) virtuality. One dramatic manner of thinking about this regime is as a strong-field configuration of QCD, a dense phase of weakly-interacting gluons. [31] It may even be possible to bring such a state into being in the laboratory;

the RHIC at Brookhaven may produce it as an initial state in nuclear collisions, as may the LHC operating with nuclear beams.

This viewpoint has been developed quantitatively through an effective theory, which has some similarities to the one described above in the context of shape functions. We introduce a set of color sources, this time coming from the distant past along the lightcone,

$$W_{\pm}(x^{\mp}, x_t) = P \exp \left[\int_{-\infty}^{\infty} dx^{\pm} A^{\mp}(x^{\mu}) \right]. \quad (24)$$

The relation of the BFKL equation to such an effective field theory was described in [32]. The nuclear connection is made by modelling a large nucleus as a distribution of the sources: [33]

$$S_{\text{nuclear field}} = S_{\text{QCD}} + \frac{i}{N_c} \int d^2 x_t dx^- \rho(x_t, x^-) W_+(x^-, x_t). \quad (25)$$

Among the intriguing results of this approach is the generation, [34] for a nucleus of essentially unlimited size, of a gluon occupation number density, which is of nonperturbative magnitude, $\mathcal{O}(1/\alpha_s)$, and which can serve as a starting point for the very complex time evolution of nucleus-nucleus collisions. [35]

5 Conclusions

Even within the area of factorization at high energy, I have of necessity passed over many developments from the past few years, regarding global PDF fits [36] and their uncertainties, [37] cross sections at measured transverse momentum, [38] diffraction, [39] higher-twist [40], polarized [41] and skewed [42] parton distributions, and more. The subject of power corrections at high energy is still new, and we are just now learning to read the quantum mechanical history of QCD scattering in the language of final states. I expect the progress of the past few years, punctuated as it is with novel ideas and applications, to continue for some time, as we ask new questions of quantum chromodynamics.

Acknowledgments

I would like to thank the organizers of the *IVth Rencontres du Vietnam* for the chance to participate in a memorable conference, and to the National Science Foundation for support. I would also like to thank Brookhaven National Laboratory for its hospitality. This work was also supported in part by the National Science Foundation, grant PHY9722101.

References

- [1] M. Klein, in *Proc. of the 19th Intl. Symp. on Photon and Lepton Interactions at High Energy LP99*, ed. J.A. Jaros and M.E. Peskin, (eConf **C990809**), p. 467 (2000); hep-ex/0001059.
- [2] D0 Collaboration (B. Abbott et al.). hep-ex/0008021.
- [3] E.W.N. Glover, 33d Rencontres de Moriond, hep-ph/9805481.

- [4] Z. Bern, L. Dixon and D.A. Kosower, *JHEP* **0001**, 027 (2000), hep-ph/0001001; E.W.N. Glover and M.E. Tejeda-Yeomans, Zeuthen Workshop on Elementary Particle Theory, 1999, *Nucl. Phys. Proc. Suppl.* **89** (2000) 196; W.L. van Neerven, 6th Workshop in High Energy Physics Phenomenology (WHEPP 6), *Paramana* **55** 101 (2000), hep-ph/0003043.
- [5] S.A. Larin, T. van Ritbergen and J.A.M. Vermaseren, *Nucl. Phys. B* **492**, 338 (1997), hep-ph/9605317.
- [6] A.L. Kataev, G. Parente and A.V. Sidorov, *Nucl. Phys. B* **573**, 405 (2000), hep-ph/9905310; W.L. van Neerven and A. Vogt, *Nucl. Phys. B* **568**, 263 (2000), hep-ph/9907472; hep-ph/0006154; J.A.M. Vermaseren and S. Moch, *Nucl. Phys. Proc. Suppl.* **89**, 131 (2000); hep-ph/0004235.
- [7] N. Kidonakis, *Int. J. Mod. Phys. A* **15**, 1245 (2000), hep-ph/9902484; S. Catani *et al.*, Workshop on Physics at TeV Colliders, Les Houches, France, 7-18 Jun 1999, hep-ph/000511.
- [8] R. Akhouri, M. Sotiropoulos and G. Sterman, *Phys. Rev. Lett.* **81**, 3819 (1998), hep-ph/9807330.
- [9] A. Vogt, hep-ph/0010146.
- [10] E.L. Berger and H. Contopanagos, *Phys. Rev. D* **57** 253 (1998), hep-ph/9706206; N. Kidonakis and R. Vogt *Phys. Rev. D* **59**, 074014 (1999), hep-ph/9806526; N. Kidonakis and J.F. Owens, *Phys. Rev. D* **61**, 094004 (2000), hep-ph/9912388.
- [11] S. Catani, M.L. Mangano, P. Nason, C. Oleari and W. Vogelsang, *JHEP* **9903**, 025 (1999).
- [12] S. Catani, L. Trentadue, G. Turnock and B.R. Webber, *Nucl. Phys. B* **407**, 3 (1993).
- [13] M. Beneke, *Phys. Rept.* **317**, 1 (1999), hep-ph/9807443; M. Beneke and V.M. Braun, hep-ph/0010208.
- [14] S.J. Brodsky, E. Gardi, G. Grunberg and J. Rathsmann, hep-ph/0002065.
- [15] G. P. Korchemsky and G. Sterman, 30th Rencontres de Moriond, hep-ph/950539; Yu.L. Dokshitzer and B.R. Webber, *Phys. Lett. B* **404**, 321 (1997), hep-ph/9704298.
- [16] Yu.L. Dokshitzer, G. Marchesini and B.R. Webber, *Nucl. Phys. B* **469**, 93 (1996), hep-ph/951233; Yu.L. Dokshitzer, A. Lucenti, G. Marchesini and G.P. Salam, *JHEP* **9805**, 003 (1998), hep-ph/9802381.
- [17] M.L. Mangano, in *High Energy Physics 99*, ed. K. Huitu *et al.* (Institute of Physics, Bristol, UK, 2000) p. 33; hep-ph/9911256.
- [18] L3 Collaboration (M. Acciarri *et al.*), *Phys. Lett. B* **489**, 65 (2000), hep-ex/0005045.
- [19] G.P. Korchemsky, 33d Rencontres de Moriond, hep-ph/9806537; G.P. Korchemsky and G. Sterman, *Nucl. Phys. B* **555**, 335 (1999), hep-ph/9902341; G.P. Korchemsky and S. Tafat, hep-ph/0007005.
- [20] G.P. Korchemsky, G. Oderda and G. Sterman, 5th International Workshop on Deep Inelastic Scattering and QCD (DIS 97), hep-ph/9708346

- [21] G.P. Korchemsky and G. Sterman, *Phys. Lett. B* **340**, 96 (1994), hep-ph/9407344; C.W. Bauer, S. Fleming and M. Luke, hep-ph/0005275.
- [22] M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, hep-ph/0006124.
- [23] Y.-Y. Keum, H.-n. Li and A.I. Sanda, hep-ph/0004004.
- [24] V.S. Fadin and L.N. Lipatov, *Phys. Lett. B* **429**, 127 (1998), hep-ph/9802290; M. Ciafaloni and G. Camici, *Phys. Lett. B* **430**, 349 (1998), hep-ph/9803389.
- [25] I. Balitsky, *Nucl. Phys. B* **463**, 99 (1996), hep-ph/9509348.
- [26] A. Sen, *Phys. Rev. D* **27**, 2997 (1983).
- [27] G.P. Salam, 39th Cracow School of Theoretical Physics, hep-ph/9910492.
- [28] S.J. Brodsky, V.S. Fadin, V.T. Kim and G.B. Pivovarov, *JETP Lett.* **70**, 155 (1999), hep-ph/9901229.
- [29] B. Andersson, G. Gustafson and J. Samuelsson, *Nucl. Phys. B* **467**, 443 (1996); C.R. Schmidt, *Phys. Rev. D* **60**, 074003 (1999), hep-ph/9901397.
- [30] M. Ciafaloni D. Colferai and G.P. Salam, *Phys. Rev. D* **60**, 114036 (1999), hep-ph/9905566; G. Altarelli, R.D. Ball and S. Forte, *Nucl. Phys. B* **575**, 313 (2000), hep-ph/9911273.
- [31] A.H. Mueller, International Summer School on Particle Production Spanning MeV and TeV Energies (Nijmegen 99), hep-ph/9911289; L. McLerran, 39th Cracow School of Theoretical Physics: Strong Interactions at Low and High Energies, *Acta Phys. Polon. B* **30**, 3707 (1999), nucl-th/9911013.
- [32] I. Balitsky, *Phys. Rev. D* **60**, 014020 (1999), hep-ph/9812311.
- [33] R. Venugopalan, 34th Rencontres de Moriond, hep-ph/9907209.
- [34] A. Krasnitz and R. Venugopalan, *Phys. Rev. Lett.* **84**, 4309 (2000), hep-ph/9909203.
- [35] R. Baier, A.H. Mueller, D. Schiff and D.T. Son hep-ph/0009237.
- [36] L. de Barbaro *et al.*, Tevatron Run II Workshop on QCD and Weak Boson Physics, hep-ph/0006300.
- [37] W. Giele and S. Keller, *Phys. Rev. D* **58**, 094023 (1998), hep-ph/980339; J. Pumplin, D.R. Stump and W.-K. Tung, hep-ph/0008191.
- [38] F. Landry, R. Brock, G. Ladinsky and C.P. Yuan, hep-ph/9905391; E. Laenen, G. Sterman and W. Vogelsang, *Phys. Rev. Lett.* **84**, 4296 (2000), hep-ph/0002078; hep-ph/0010080; S. Catani, D. de Florian and M. Grazzini, hep-ph/0008184.
- [39] J.C. Collins, *Phys. Rev. D* **57**, 3051 (1998); **E**, *ibid.* **D 61**, 019902 (2000), hep-ph/9709499; L. Alvero, J.C. Collins, J. Terron and J.J. Whitmore, *Phys. Rev. D* **59**, 074022 (1999), hep-ph/980526.

- [40] V.M. Braun, G.P. Korchemsky and A.N. Manashov, *Phys. Lett. B* **476**, 455 (2000), hep-ph/0001130; R.J. Fries, A. Schafer, E. Stein and B. Müller, *Nucl. Phys. B* **582**, 537 (2000), hep-ph/000207; X.-f. Guo and X.-N. Wang hep-ph/0005044; S. Brodsky, Workshop on Nuclear Structure in High x-Bjorken Region (HIX2000), hep-ph/0006310; V.M. Braun, R.J. Fries, N. Mahnke and E. Stein, hep-ph/0007279.
- [41] E.W. Hughes and R. Voss, *Ann. Rev. Nucl. Part. Sci.* **49**, 303 (1999); A.V. Belitsky, X.-d. Ji, W. Lu and J. Osborne, hep-ph/0007305; V.M. Braun, G.P. Korchemsky and A.N. Manashov, hep-ph/0010128.
- [42] C. Ji, *J. Phys. G* **24**, 1181 (1998), hep-ph/980735; A.V. Belitsky, A. Freund, and D. Müller *Nucl. Phys. B* **574**, 347 (2000), hep-ph/9912379; A.V. Radyushkin, *Nucl. Phys. A* **666**, 225 (2000).